from all observations. In the extreme, the difference between the coefficients based on early and late points is as much as 0.3% even though the average deviation of the point by point rate coefficients based on a given set of parameters is of the order of 0.03%. Thus it cannot be concluded that rate data are correct only on the basis of a lack of trend in the point by point rate coefficients.

One of the time consuming aspects of this technique is the independent determination of the conductance parameters Λ_0 and S_{α} . As one might expect, the parameters derived from the rate data are not very sensitive to the value of Λ_0 but they are much more sensitive to the value of S_{α} , the limiting slope of the Λ vs. $c^{1/2}$ plot. It is primarily S_{α} which determines the correction to the rate data as is evident from equation 4 obtained by rearranging equation 3 (see Experimental).

$$10^{3} K/Rc = \Lambda_{0} - S_{\alpha}c^{1/2}$$
⁽⁴⁾

If the left hand side of (4) were a constant then obviously no correction would be necessary. If data of the optimum precision are not required one can often estimate the value of Λ_0 from data in the literature and calculate S_{α} from Onsager's relation.³¹ We used this procedure to treat the rate data obtained in 50% ethanol, but one cannot use this approach unless ion association is negligible. It seems feasible to derive the conductance values from the rate data but we have not pursued this.

Table IV gives a comparison of the rate constants obtained by the conductometric technique with those determined titrimetrically by Fainberg and Winstein.^{32,33} The reliability of the titrimetric values is estimated to be 1-2%. The conductometric values are all higher than the titrimetric

(31) H. S. Harned and B. B. Owen, "Electrolytic Solutions," Third Edition, Reinhold Publishing Corp., New York, N. Y., 1958.
(32) A. H. Fainberg and S. Winstein, J. Am. Chem. Soc., 78, 2770

(1956). (33) A. H. Fainberg and S. Winstein, *ibid.*, **79**, 1597 (1957). values.¹⁴ With *t*-butyl chloride the agreement is probably within the experimental error; but in the case of 1-phenylethyl chloride, while there is good agreement between the values in 80%ethanol, the difference increases in the more aqueous solvents. Since these two compounds solvolyze at comparable rates, the discrepancies cannot be due to some error which is proportional to the rate. The difficulty probably is due to the decreasing solubility of 1-phenylethyl chloride in the more aqueous solvents. The concentration of alkyl halide used by Fainberg and Winstein was from ten to thirty-five times that in our conductometric determinations.

To summarize, we have applied the techniques of precise conductance measurements to the problem of rate determination. The reproducibility study of the solvolysis of 1-phenylethyl chloride indicates that the rate coefficient can be determined with a precision of 0.02% if one exercises the utmost care and carries out a number of runs. In the absence of great care the limits of reproducibility may rise to 0.1% but no higher in our experience if the solvent and compound are pure. It is expected that, with further experience with this method, improvements will be forthcoming in defining more precisely the factors governing precision, if not in the precision itself.

Acknowledgment.—The authors wish to express their appreciation to Mr. Earl Sexton for his patience and attention to the difficult problem of constructing the conductance cells and to Miss Joanna Dickey for frequent weighings. It is a pleasure to acknowledge many valuable discussions with Dr. E. J. Bair. The facilities of the Indiana University Research Computing Center were used for the statistical computations.

(34) Often rate constants determined conductometrically are higher than the titrimetric values. See W. M. Schubert and R. G. Minton, *ibid.*, **82**, 6188 (1960).

[CONTRIBUTION FROM THE DEPARTMENT OF CHEMISTRY, UNIVERSITY OF CALIFORNIA, LOS ANGELES 24, CALIFORNIA] Proton Exchange Rates and Hydrogen-Bonding for Water in Organic Solvents¹⁸

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The chemical behavior of water protons in a series of organic solvents has been studied by nuclear magnetic resonance and infrared techniques. The shifts in the OH stretching frequencies and the n.m.r. chemical shifts of the water protons have been measured in various solvents and correlated with each other, and with hydrogen bond energies. A theory of chemical shifts due to hydrogen bonding is discussed. The equilibrium constants for water dimer formation in these solvents have been estimated from the n.m.r. data. The n.m.r. spectral line shape has been studied as a function of solvent, temperature, water concentration, and added acid, for dilute solutions of H_2O-D_2O mixtures with a hydrogen to deuterium ratio of one. The rate of inter-water proton exchange was determined by this technique. At room temperature, the mean exchange rate τ^{-1} for 1.1 formal water in various solvents is (in sec. ⁻¹): 6.7 (nitromethane), 0.91 (acetonitrile), 1.0 (acetone), 1.6 (dioxane), 25 (pyridine), 8.3 (dimethylsulfoxide), 100 (triethylamine). The apparent activation energy is less that 1.5 kcal./mole for the weakly basic solvents and the reaction is approximately second order in water. The dominant mechanism in the first five solvents is thought to be a direct proton exchange rate increasing temperature; the net result is an exchange rate which is almost temperature independent. The trimer concentrations in dimethylsulfoxide and triethylamine are too low to account for the observed rates but the concentrations of OH⁻ and protonated solvent SH⁺ are high in these two solvents and, hence, mechanisms that depend upon OH⁻ and SH⁺ can become important. The activation energy for the process involving OH⁻ is probably lower than for the one in which SH⁺ partakes; therefore, the OH⁻ mechanism

Introduction

Very little is known about the behavior of water as a solute rather than as a solvent. This study

(1) (a) Supported in part by the Research Corporation and pre-

represents a preliminary investigation of water dis-

sented in part at the Southern California Regional Meeting of the American Chemical Society, Los Angeles, December, 1960. (b) Alfred P. Sloan Fellow. (c) J. R. Holmes, D. Kivelson and W. C. Drinkard, J. Chem. Phys., 37, 150 (1962). TABLE I

			OBREAL OU	AIMAAI			
Solvent	Δ _{νs} , cm. ⁻¹	ΔH_{\min}^{∞} , kcal./mole	– RT in K _x , kcal./mole	<i>E</i> _H − <i>E</i> _H (H ₂ O), kcal./mole	75 ⁻¹ . sec1	τ4 ^{~1} , sec. ^{−1}	Exp. τ^{-1} , sec. τ^{-1}
Carbon tetrachloride	40						
Benzene	75						
Nitromethane	78	2.4^{b}	-1.2	-1.6	3.1		6.8
Acetonitrile	115	1.89°	-0.7	-1.4	0.9		0.9
Acetone	127	1.05^{d}	5	-0.9	(1.0)		1.0
<i>p</i> -Dioxane	153	1.6°	- .6	-1.2	1.7		1.6
Tetrahydrofuran	196^a		7				
Water	276						
Ethyl acetate							11
Pyridine	292	-0.65^{e}	-1.0	-0.1	3.6	2.2	25
Dimethyl sulfoxide	334	-1.3'	0.2	0.2	0.18	6×10^4	8.3
Trimethylamine	358	-1.6^{g}		0.3		≫100	>100
a Measured in the pres	ent work.	all other An	volues from re	f 9 b Dof	6 CDof 7	d Dof 9 d Dof	

^a Measured in the present work; all other $\Delta \nu_0$ values from ref. 3. ^b Ref. 6. ^c Ref. 7. ^d Ref. 8. ^e Ref. 9. ^f Ref. 10. ^g Ref. 11.

solved in a series of organic solvents, all of which are proton acceptors. Proton magnetic resonance has been the principal tool used in this research but useful data also have been gathered from infrared and calorimetric studies. In the first few sections of this article the thermodynamic properties of the "dilute water solutions" are discussed. Heats of mixing are related to hydrogen bond energies, to infrared O-H stretching frequency shifts, and to chemical shifts in the proton resonance spectra; polymerization constants are obtained from chemical shift data, and the concentrations of ionic species are estimated by a variety of techniques. In the latter sections of the article the proton resonance spectral pattern, which has been discussed elsewhere,^{1c} has been used to study the kinetics of proton exchange for water in various solvents. The results of this investigation are rather tentative since the relevant experimental data are still fragmentary, but it is hoped that the approach suggested will be fruitful.

Infrared Shifts. – The average infrared stretching frequency for the O–H bond in water vapor is about 3700 cm.⁻¹; hydrogen bonding causes a shift to lower frequency and an increase in the width and intensity of the absorption band.² This infrared shift $\Delta \nu_s$ can be measured relative to the position of of the band in water vapor and is a useful measure of hydrogen bonding. The measurements have been made on dilute solutions of water (0.01 to 0.05 formal). Under these conditions the spectra show only the "associated bands" attributed to the hydrogen-bonded complex of water and solvent.² Values of infrared shifts measured by others^{3,4} and in the course of this work are presented in Table I.

Heats of Mixing and Hydrogen Bonding Energies.—The enthalpy change of mixing ΔH_{mix} for water dissolved in proton-accepting solvents is abnormally large, primarily because hydrogen bonds are broken and formed in the solution process. The process may be endothermic, as with

(2) G. C. Pimentel and A. L. McClellan, "The Hydrogen Bond," W. H. Freeman and Co., San Francisco, Calif., 1960.

(3) E. Greinacher, W. Luttke and R. Mecke, Z. Elektrochem., 59, 23 (1955).

(4) P. Saumagne and M. L. Josien, Bull. Soc. Chim. (France), 25, 813 (1958). The values reported by these workers differ slightly from those given in ref. 3, but the discrepancies do not affect any of the present conclusions. The values from ref. 3 are quoted in Table I.

"inert" solvents, or exothermic, as with solvents which are more basic than water.⁵ Several values of $\Delta H_{\min} \infty$, the heat of mixing per mole of water extrapolated to infinite dilution, are given in Table I.⁶⁻¹¹ Hydrogen bonds do not play an important role in the solvent-solvent interactions for the liquids studied here. If one assumes that the non-hydrogen bonding interactions do not contribute to the heat of mixing, a reasonable approximation since these interactions are small and probably about the same for all interacting molecular pairs, then $\Delta H_{\rm mix}^{\circ\circ}$ is the energy difference, per mole of water, between hydrogen bonding interactions of water with water and those of water with solvent. Haggis, et al.,12 have estimated the energy of the water-water hydrogen bond at 4.4 kcal. per mole and the percentage of possible O-H hydrogen bonds formed in pure water at ninety. The effective water-water hydrogen bond energy in pure water is, therefore, 4.0 kcal. per mole. If one assumes that water at infinite dilution in the various solvents considered here exists principally as the hydrogen-bonded complex consisting of one water molecule and two donor molecules, the energy $E_{\rm H}$ of a single water-solvent hydrogen bond may be estimated by the relation

 $E_{\rm H} = (4.0 - \Delta H_{\rm mix}^{\infty}/2) \, \rm kcal./mole$ (1) In Fig. 1 the infrared shifts are plotted against values of $E_{\rm H}$ estimated in this way; a reasonably good straight line results, amazingly good when one considers that the calorimetric measurements are not very accurate and that the theory is very approximate. The assumption that the infrared shift is a linear measure of the hydrogen bonding energy² seems to be borne out by these results. It should be mentioned that other estimates, drastically dif-

(5) J. S. Rowlinson, "Liquids and Liquid Mixtures," Butterworths Scientific Publications, London, 1959.

(6) D. B. Myers, Department of Chemistry, UCLA, private communication.

(7) A. L. Vierk, Z. anorg. chem., 261, 283 (1950).

(8) W. C. Drinkard and D. Kivelson, J. Phys. Chem., 62, 1494 (1958).

(9) R. J. L. Andon, J. D. Cox and E. F. G. Herrington, Trans. Faraday Soc., 53, 410 (1953).

(10) J. Keuttämaa and J. J. Kindberg, Suomen Kemistilehti, 33B, 32 (1960).

(11) I. L. Copp and D. H. Everett, Discussions Faraday Soc., 15, 174 (1953).

(12) G. H. Haggis, J. B. Hasted and T. J. Buehanan, J. Chem. Phys., **80**, 1452 (1952).



ferent from that of Haggis, *et al.*¹² have been made for $E_{\rm H}({\rm H_2O})$ and for the percentage of possible hydrogen bonds in water. If different values for these two quantities are used, the slope of the line in Fig. 1 remains unchanged but the $E_{\rm H}$ intercept is altered.

Proton Magnetic Resonance Chemical Shifts.— Chemical shifts, relative to those for pure H₂O, have been measured at 40 Mc. for H₂O dissolved in a number of organic solvents. The mole fractions of H₂O ranged from 0.1 to 0.002 (see Fig. 2). The dependence upon H₂O concentration was linear within experimental error and the chemical shifts at infinite dilution $\Delta \nu_{0}^{0}$ were obtained by extrapolation. By means of standard calculations, corrections for bulk diamagnetic effects were made wherever volume diamagnetic susceptibilities of the solvent were known.¹³ The susceptibility of

TABLE II

DATA FROM N.M.R.

(H₀O);

Solvent	Δν ⁰ οbs, c.p.s,	Δνο, c.p.s.	∂(Δ ν)/ ∂X _{H2O} , c.p.s.	Kx	K _m , 1./mole	concn, at 1.1 formal H ₂ O, moles/1.
Carbon tetra-						
chloride		150°				
Benzene		172^{b}				
Nitromethane	139.9	106.5	435	6.8	0.37	0.058
Acetonitrile	119.3	118.0	206	3.4	. 18	.017
Acetone	105.2	83.3	122	2.3	.17	.018
Tetrahydro-						
furan	88.0^{a}	86.5	195	3.6		
Dioxane	97.6	88.0	156	2.9	.25	.030
Dimethyl						
sulfoxide	60.5	50.4	25	0.7	. 56	.003
Pyridine	28.1	18.7	113	5.6	. 45	. 062
Triethylamine	71.5	58.4				
-						

^a Corrected for bulk susceptibility by means of internal standard. , ^b Taken from the work of Cohen and Reid.¹⁵

tetrahydrofuran is not known and the H₂O proton resonance was measured relative to the β proton resonances of tetrahydrofuran both in solutions of water in tetrahydrofuran and in solutions of tetra-

(13) J. A. Pople, W. G. Schneider and H. J. Bernstein, "High Resolution Nuclear Magnetic Resonance," McGraw-Hill Book Co., Inc., New York, N. Y., 1959.



hydrofuran in water. In these measurements it was assumed that the tetrahydrofuran resonance is not subject to any solvent effects other than the bulk diamagnetic effect, and, consequently, that it can be used as an internal standard. The two methods for correcting for bulk diamagnetic effects were

TABLE III ESTIMATES OF IONIC CONCENTRATION Concentrations for 1.1 formal H₂O (OH ⁻) (H₄O ⁺) (SH ⁺)

Solvent	e, e.s.u.	$K_{\rm W} \times 10^{20}$	$\times 10^{10}$	$\times 10^{10}$	$\times 10^{10}$
Nitro-					
methane	39.4	20	5	5	0.11
Aceto-					
nitrile	38.8	20	5	4	1.2
Acetone	21.4	0.2	0.8	0.3	0.5
Dioxane	2.2	0.9^{a}	1.2	0.9	0.25
Pyridine	12.5	0.0013	120		120
Dimethyl					
sulfoxide	46.4	50	4×10	6	$4 \times 10^{\circ}$
Water	78.5	324			

^e From data of Harned and Fallon, ref. 29.

checked for p-dioxane and acetone; the corrections evaluated by the different methods agreed to within 0.4 c.p.s. For further experimental details see ref. 14. Values of $\Delta \nu_0$, the chemical shift corrected for bulk diamagnetic effects, are presented in Table II; some of the data were taken from the work of Cohen and Reid.¹⁵

It is convenient to express the chemical shifts relative to that of water vapor, in which there is, presumably, negligible hydrogen-bonding interaction. The chemical shift of water vapor relative to that of liquid water, corrected for the bulk

(14) J. R. Holmes, Doctoral Thesis, University of California, Los Angeles 24, California, 1961.

(15) H. D. Cohen and C. Reid, J. Chem. Phys., 25, 791 (1956).



Fig. 3. $-\Delta \nu_{\rm H}$ vs. ν_{s} : $\Delta \nu_{\rm H}$ for CCl₄ and C₆H₅ are from ref. 15; measured value of $\Delta \nu_{\rm H} = 183$ c.p.s. for H₂O.

susceptibility of liquid water, is 183.2 c.p.s.¹⁶ The chemical shifts $\Delta v_{\rm H}$, relative to that of water vapor, of water at infinite dilution in organic solvents, are given in Fig. 3. The chemical shifts $\Delta v_{\rm H}$ contain contributions from local diamagnetic anisotropies of the donor molecule in the region of the hydrogen bond site. These contributions are difficult to calculate; however, they have been estimated for a proton bonded to the π -cloud of benzene^{13,14} and to the triple bond of acetylene.14.16 If the triple bond in acetonitrile is treated analogously to that in acetylene, one would expect an anisotropic shift of about -25 c.p.s. for water protons bonded to acetonitrile. Acetone, nitromethane, p-dioxane tetrahydrofuran and dimethylsulfoxide should exhibit similar but smaller shifts. The estimated corrections in $\Delta \nu_{\rm H}$, arising from the anisotropic shifts, are indicated by the arrows in Fig. 3, a figure which represents a plot of the chemical shifts $\Delta \nu_{\rm H}$ against the infrared shifts $\Delta v_{\rm B}$. The $\Delta v_{\rm H}$ for water has been corrected for the fact that 10% of the possible hydrogen bonds in liquid water are dissociated12; if another estimate of this percentage is taken the $\Delta \nu_{\rm H}$ will be shifted accordingly.

Figure 3 indicates that for the weakly basic solvents the shielding decreases as the hydrogen bond strength increases, while for the strongly basic solvents the shielding increases as the hydrogen bond strength increases. Mavel¹⁷ has proposed that the sign reversal of $(\partial \Delta \nu_{\rm H} / \partial \Delta \nu_{\rm S})$ for strongly basic solvents is a consequence of the protolysis reaction

$$H_2O + S \longrightarrow SH^+ + OH^- \qquad K_B$$
 (2)

where S represents a solvent molecule and $K_{\rm B}$ the equilibrium constant. The observed proton resonance would be an average of that for S \cdots H₂O, SH⁺ and OH⁻, and the chemical shift would be displaced towards higher fields relative to solvated water, S \cdots H₂O.¹⁸ The O-H infrared shift is, of course, a measure of the single species S \cdots H₂O and would not be much affected by the protolysis reaction, but the heats of mixing and the hydrogenbond energies $E_{\rm H}$ should be. No anomalous be-

(16) W. G. Schneider, H. J. Bernstein and J. A. Pople, J. Chem. Phys. 28, 1487 (1958).

(17) G. Mavel, (a) Compt. rend., 248, 1505 (1959); (b) *ibid.*, 249, 1753 (1959); (c) *J. chim. phys.*, 57, 649 (1960). Mavel actually considered $\partial \Delta_{\nu H}/\partial (K_B/K_W)$, where K_B and K_W apply to aqueous solutions.

(18) C. MacLean and E. L. Machor, J. Chem. Phys., 34, 2207 (1961).

havior is observed for $E_{\rm H}$ in the strongly basic solvents (see Fig. 1). Furthermore, crude estimates of SH⁺ and OH⁻ concentrations, even in solvents as basic as dimethylsulfoxide, indicate that these ions cannot contribute appreciably to the chemical shift (see Table III). Mavel's estimates of ionic concentrations probably are quite high since his $K_{\rm B}$'s were evaluated for S in H₂O rather than for H_2O in S; the dielectric constant for the latter solution is very much smaller than for the former and the effective $K_{\mathbf{B}}$ will, consequently, be greatly reduced. This is discussed in a later section. Furthermore, recent evidence suggests that the presence of OH - ions would cause a low field shift.¹⁹ The evidence presented here suggests the need of an alternative explanation for the sign of $(\partial \Delta \nu_{\rm H} / \partial \Delta \nu_{\rm S})$ in strongly basic solvents.

Marshall, Pople and Buckingham²⁰ have developed a theory which allows one to estimate changes in shielding resulting from the inhibition by an applied electric field of the diamagnetic circulation of electrons about a proton. This shielding is a "paramagnetic" term, similar to the paramagnetic term in Ramsey's expression for dia-magnetic shielding,²¹ and has a r^{-3} dependence where r is the electronic-nuclear distance. The decrease in shielding varies as the square of the applied electric field. In a simple point charge electrostatic theory for hydrogen bonding, the electric field due to the hydrogen bond is proportional to $E_{\rm H}$ and the reduction of the shielding should then be proportional to $E_{\rm H}^2$. But in a simple covalent theory of the hydrogen bond the electron density about the proton increases² with increasing $E_{\rm H}$ and the diamagnetic r^{-1} terms in Ramsey's expression for the shielding²¹ increase, thereby increasing the shielding. The electrostatic theory can, therefore, be used to estimate the paramagnetic contribution to the shielding whereas the covalent theory can be used to approximate the diamagnetic contribution. In keeping with this viewpoint the hydrogen bond wave function can be written as $\alpha_e \varphi_e + \alpha_c \varphi_c$ where the subscripts e and c represent the electrostatic structure S. . . HO and the covalent structure (S $\cdot \cdot H^+$ O⁻), respectively. In this description it would appear that in all systems studied here the electrostatic structure dominates but that in the solvents which are more basic than water the covalent structure becomes important. Of course, another way of describing the variations in shielding with changes in $E_{\rm H}$ is to say that the variations in the paramagnetic terms dominate but that the variations in the diamagnetic terms become appreciable for strongly basic solvents.

Equilibrium Constants for the Polymerization of Water.—Chemical shift measurements can be used to obtain information about polymerization.²²⁻²⁴

(19) J. I. Musher, ibid., 35, 1989 (1961).

(20) T. W. Marshall and J. A. Pople, Mol. Phys., 1, 199 (1958); A. O. Buckingham and J. A. Pople, Proc. Camb. Phil. Soc., 53, 262 (1957).

(21) N. F. Ramsey, "Nuclear Moments," John Wiley and Sons, New York, N. Y., 1953, p. 273.

(22) C. M. Huggins, G. C. Pimentel and J. N. Shoolery, J. Chem. Phys., 23, 1244 (1956).

(23) S. K. Alley, Doctoral Thesis (Chemistry), University of Callfornia, Los Angeles, 1961. It is not practicable to attempt an analysis over the entire mole fraction range of water but in terms of a simplified model the low water concentration region may be studied profitably. In very dilute solutions only two species will be considered: the monomer complex MS₂ represented as

$$\checkmark_{H\cdots S}^{H\cdots S}$$

and the water dimer M2S2 represented as

$$s \cdots H$$
 $o \cdots H - o$ $H \cdots h$

The two species are considered to be in equilibrium according to the relation

$$2MS_2 \xrightarrow{} M_2S_3 + S \qquad (3)$$

The MS₂ protons have a shift given by the value of $\Delta \nu_{\rm H}$ discussed above. The M₂S₃ protons have an average shift of $(3\Delta \nu_{\rm H} + \Delta \nu_{\rm H,0})/4$ where $\Delta \nu_{\rm H,0} = 204$ c.p.s., the corrected chemical shift of liquid water. Because of the rapid reorientation of the molecules in solution, the observed chemical shift $\Delta \nu$ for all water protons involved in the equilibrium is an average of the individual shifts

$$\Delta \mathbf{r} = (n_{\rm m}/n_{\rm w})(\Delta \mathbf{r}_{\rm H}) + 2(n_{\rm d}/n_{\rm w})(3\Delta \mathbf{r}_{\rm H} + \Delta \mathbf{r}_{\rm H10})/4 \quad (4)$$

where n_m and n_d are the number of moles of monomer and dimer, respectively, and n_w is the number of moles of water present, *i.e.*, $n_m + 2n_d$. The mole fraction equilibrium constant K_x for the dimerization in eq. (3) can be expressed as^{18,22–24}

$$K_{\mathbf{x}} = 2[\partial(\Delta \mathbf{r})/\partial(n_{\mathbf{w}}/n_{\mathbf{s}})]_0/(\Delta \mathbf{r}_{\mathrm{H}} - \Delta \mathbf{r}_{\mathrm{H},0}) \qquad (5)$$

where $n_{\rm s}$ is the number of moles of solvent, the derivatives are evaluated from Fig. 2 by extrapolating to $(n_{\rm w}/n_{\rm s}) \rightarrow 0$, and the values of $\Delta v_{\rm H}$ are taken from Table I. The limiting values of the slopes, the resulting values of $K_{\rm x}$, and those of the molar equilibrium constants $K_{\rm m}$ are given in Table II.

equilibrium constants $K_{\rm m}$ are given in Table II. The most significant error in the experimental evaluation of $K_{\rm x}$, provided the various assumptions are justified, is the uncertainty in the determination of the slope. This uncertainty depends upon the scatter of the chemical shift measurements over the range of shifts measured and is estimated to be about $\pm 20\%$. The value of $K_{\rm x}$ for pyridine is subject to another serious uncertainty; $(\Delta \nu_{\rm H} - \Delta \nu_{\rm HsO})/4 = 10$ c.p.s. but if the highly uncertain 10% correction for the broken hydrogen bonds in pure water is not made, $(\Delta \nu_{\rm H} - \Delta \nu_{\rm HsO})/4 = 5$ c.ps. which alters $K_{\rm x}$ by 100%.

The free energies, $-RT \ln K_x$, for the dimerization reaction of eq. 3 at 300°K., are given in Table I. Since dimerization involves the breaking of ordered S···H bonds and the formation of ordered O···H bonds, one would expect the entropy change to be reasonably small and the free energy change to be approximately equal to the change in enthalpy. This latter quantity is approximately $E_{\rm H} - E_{\rm H}$ (water), values of which are listed in Table I. These values are, of course, sensitive to the choice of the effective energy of the hydrogen bond in water whereas the linear relation illustrated in Fig. 1 is not. The values for the free energy and en-

(24) B. D. Becker, V. Liddel and J. N. Shoolery, J. Mol. Spec., 2, 1 (1958).

thalpy changes given in Table I imply small negative entropy changes for most of the systems studied; the positive entropy change for the pyridine system is subject to great uncertainty as discussed above. These conclusions are, of course, tentative; the determination of K_x as a function of temperature would give an independent check on these results.

The determination of K_x and K_m was carried out at very low water concentrations. As the water concentration is increased trimers and higher polymers tend to form. The equilibrium constant $K_{\mathbf{m}}$ for the formation of trimer from monomer and dimer is probably of the same order of magnitude but somewhat smaller than K_m . Approximate trimer concentrations, assuming $K_m = K_m'$ are given in Table II for 1.1 formal water concentrations: the true trimer concentration is probably somewhat smaller. For water concentrations above 0.5 formal in the solvents studied, the trimer concentration depends very nearly on the square of the formal water concentration rather than on the third power as it would for much lower concentrations. Mavel has estimated the equilibrium constants for formation of trimer from three monomer molecules as 6×10^{-3} and 1.5×10^{-2} for water in acetone and dioxane, respectively^{17a}; these should be compared with the $\bar{K}^2_{\rm m}$ values of 2.9 \times 10⁻² and 6.2×10^{-2} , respectively, that can be obtained for these compounds from Table II.

Under the assumption, discussed above, of small entropy change for water polymerization in the solvents studied, the enthalpy should lie between 0 and -2.5 kcal./mole. This conclusion will enter into the discussion of the kinetics of proton exchange.

Ionic Concentrations.—The ionic species H_3O^+ , OH⁻ and SH⁺ exist in the various solutions studied, but it is not a simple matter to determine the concentrations of these ions in dilute solutions of water in organic solvents. The following estimates of ionic concentrations are extremely uncertain; nevertheless, they are useful, in the absence of better data, in discussing the mechanisms of proton exchange. The ions arise from the reaction in eq. 2 and from the reaction

$$2H_2O = H_3O^+ + OH^- \qquad K_W \quad (6)$$

where K_W is the equilibrium constant. If K_B and K_W are expressed in terms of concentrations rather than activities they are not truly constant over the entire range of water concentrations; in fact, they depend markedly upon dielectric constant and one should write K(aq) and K(s) for the equilibrium constants in solutions dilute in organic liquids and dilute in water, respectively.

The quantity $K_{\rm B}({\rm aq})$ is a measure of the basicity of the organic liquid and it has been measured for a number of substances diluted in aqueous medium. Gordy and Stanford²⁵ have found a reasonably good correlation between infrared shifts $\Delta \nu_{\rm s}$ and the $pK_{\rm B}({\rm aq})$. This relationship can be partially understood if one assumes that the entropy change for reaction 2 is negligible and, therefore, that the difference in proton affinity of the substance S and the OH⁻ ion is approximately $-2.3RTpK_{\rm B}({\rm aq})$.

(25) W. Gordy and S. C. Stanford, J. Chem. Phys., 8, 170 (1940).



Fig. 4.— $p(K_W/K_B)$ vs. $\Delta \nu_s$; $p(K_W/K_B)$ for $(CH_3)_2SO_3$ are from ref. 17; for C_8H_8N from ref. 27, and for all others from ref. 2b.

The linear relationship found by Gordy and Stanford then suggests that the strength of the hydrogen bonding interaction, measured by $\Delta \nu_s$, is proportional to the proton affinity. In the present experiments, carried out with dilute solutions of water in organic solvents, the basicity is determined by the equilibrium constant $K_{\rm B}({\rm s})$, which has not been measured for most solvents and differs greatly from $K_{\rm B}({\rm aq})$, largely because of the dependence of $K_{\rm B}({\rm s})$ upon dielectric constant. No sensible correlation, therefore, would be expected between $\Delta \nu_{\rm s}$ and $pK_{\rm B}({\rm s})$ since the proton affinity of OHdepends markedly upon dielectric constant and hence upon solvent.

The reaction

$$SH^+ + H_2O \xrightarrow{\longrightarrow} H_3O^+ + S \tag{7}$$

has an equilibrium constant (K_W/K_B) . If K_W and $K_{\rm B}$ depend similarly upon the dielectric properties of the solvent, an assumption which is implicit in Eq. 8 below, then K_W/K_B should be only slightly dependent upon solvent and, hence, more or less independent of the concentration ratio of non-ionic species, $(H_2O)/(S)$. The ratio $(H_3O^+)/(SH^+)$ can be estimated reasonably accurately in the various solvents in the presence of strong reference acids. Values of $p(K_W/K_B)$ evaluated from $(H_3O^+)/(SH^+)$ measurements^{17,26,27} are plotted against the infrared shifts in Fig. 4. The value for dimethyl sulfoxide has been determined by means of Mavel's plot of $p(K_W/K_B)$ versus Δv_H and the value of $\Delta \nu_{\rm H}$ given in Fig. 3.^{17b} The only point which lies badly off the line in Fig. 4 is that for dioxane. Since dioxane has two hydrogen bonding units whereas the other solvents each have one, the quantity $p(K_W/2K_B)$ for dioxane should replace its $p(K_W/K_B)$ value in Fig. 4, provided (S)>>(SH⁺); this would improve the linear relationship somewhat. If one assumes that the entropy change for reaction 7 is negligible, then $-2.3 R T p(K_W/K_B)$ is approximately equal to the difference in proton affinities of H₂O and S, and the linear curve in Fig. 4 again suggests that the proton affinity is proportional to the strength of the hydrogen bonding interaction. Mavel's correlations between $\Delta \nu_{H}$ and

 $p(K_W/K_B)$ can be understood in terms of Figs. 3 and 4, and the discussion below eq. 2.

In order to determine ionic concentrations either $K_{\rm W}({\rm s})$ or $K_{\rm B}({\rm s})$ must be known in addition to $K_{\rm W}/K_{\rm B}$. No good values of either $K_{\rm W}({\rm s})$ or $K_{\rm B}({\rm s})$ are available but, in the absence of a better approach, $K_{\rm W}({\rm s})$ may be approximated by the relation²⁸

$$\frac{\mathrm{d}\,\ln\,K_{\mathrm{W}}}{\mathrm{d}(1/\epsilon)} = \frac{Z_1 Z_2 e^2}{(r_1 + r_2)kT} \tag{8}$$

where Z_1e and Z_2e are the charges on the ions, $r_1 + r_2$ the distance between charge centers in the undissociated state, and ϵ the effective dielectric constant. For the dissociation of H₂O, the H₃O⁺...-OH⁻ distance is approximately 2.5 Å. This formula, applicable for $\epsilon > 20$, has been applied to a number of solvents, and the resulting Kw's are give in Table III. This procedure is not satisfactory for substances with low dielectric constants such as p-dioxane ($\epsilon = 2.2$). Harned and Fallon²⁹ have made direct measurements on Kw in waterdioxane mixtures containing up to 70 mole per cent. dioxane. Extrapolation of these data to 1.1 mole/ liter of water gives $K_W(s) = 8.6 \times 10^{-21}$. Pyridine has a small dielectric constant, 12.5, and this value of $K_W(s)$ is particularly subject to error.

The ionic concentrations have been estimated from these results and are given in Table III. Although not much confidence can be placed in the accuracy of these figures, they represent general trends that are useful in analyzing the kinetic data presented below.

Kinetic Data.—For very slow proton exchange the proton resonance spectra of H_2O-D_2O mixtures in organic solvents consist of the HDO triplet and the H_2O singlet (see Fig. 1 in ref. 1c). If τ is the mean life of a proton on a particular water molecule, the line shapes can be calculated as a function of τ , the H–D spin–spin coupling constant A_{HD} and the relative chemical shift δ_{HD} between HDO and H₂O. These calculations, based upon the theories of Sack³⁰ and Anderson,³¹ are discussed in the Appendix. Some theoretical proton resonance spectra at 40 Mc., calculated for HDO-H₂O mixtures with a H/D ratio of unity and for $\tau/T_2 << 1$, where T_2^{-1} is the line width in the absence of exchange, are illustrated in Fig. 5.

The appearance of the H₂O-HDO multiplet at a given water concentration shows a marked dependence on the nature of the solvent. The spectra for solutions with a 1.1 formal water concentration in various solvents and a H/D ratio of unity are shown in Fig. 6. The values of τ^{-1} derived from the spectra are compiled in Table I. The τ 's were measured as a function of total water concentration for concentrations ranging from 0.55 to 5.5 molar; τ^{-1} seems to be approximately linear in water concentration. Since the proton exchange rates for 1.1 molar solutions in acetonitrile and acetone are too slow to affect the line shapes much, these values of τ^{-1} had to be obtained from more concentrated

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Fig. 5.—-Theoretical rate-dependent spectra; peak areas not normalized.

Fig. 6.--Experimental solvent-dependent spectra.

solutions and the assumption of a linear dependence of τ^{-1} upon the water concentration.

Although the measurement of τ is not always straightforward, particularly in spectra where the natural line widths T_2^{-1} and field inhomogeneity are comparable to the exchange broadening, the data for τ are probably good to within $\pm 25\%$ in the range $\tau = 0.6$ to 0.2 second. The rate of exchange in pyridine is too fast to estimate τ reliably by a direct comparison of line shapes. But there is a detectable broadening of the H₂O-HDO line in pyridine compared with the line observed for pure H₂O in pyridine at the same concentration; this broadening has been attributed to exchange.

The temperature dependence of τ has been measured over a 50 to 80° range in nitromethane, acetonitrile, acetone and dioxane. Within experimental uncertainty no change in τ was observed; one can therefore conclude that the effective activation energy for the exchange process is very low. At temperatures of -20° in acetone the HDO triplet was broadened whereas the H₂O line did not seem to be. This broadening might be attributed to deuterium quadrupole relaxation which becomes more important at high viscosity, or to a slightly negative "effective activation energy" of hydrogen exchange arising, as explained below, from a negative enthalpy of trimer formation.

Small amounts of hydrochloric or perchloric acid catalyzed the proton exchange of the H₂O-HDO mixtures in acetone, but the results were not reproducible. In order to prevent the leaching out of alkali from the glass Pyrex equipment and glassware coated with Desicote silicone films were used in some experiments. No improvement was noted. In acetone the HCl apparently adds to mesityl oxide, a material which forms by a condensation reaction, accelerated by the presence of water, and which is normally present in trace amounts even in freshly purified acetone.³² All solvents used in this work were highly purified; details are given elsewhere.¹⁴

Exchange rates in solutions 1.1 formal in water were greatly increased ($\tau < 0.01 \text{ sec.}$) by added acid concentrations above 10^{-6} formal. At acid concentrations below 10^{-8} formal the exchange rate seemed unaffected; at acid concentrations between 10^{-6} and 10^{-8} formal the catalytic effect was noticeable but the reproducibility was poor. With time the acid was "consumed" and the exchange rate returned to normal; 10^{-5} formal added acid had a marked effect on the exchange rate, but the effect disappeared slowly over a period of about six hours.

Throughout this work it is assumed that no significant kinetic isotope effect operates in the systems under study. In principle an isotope effect could be determined by a careful analysis of line shapes as a function of (H)/(D) ratios, although in practice the precision obtained in measuring τ from line shapes is probably not adequate; this is discussed further in ref. 14.

Mechanism of the Proton Exchange Reaction.— The mechanisms for the proton transfer reactions discussed in the preceding section have not been worked out completely but some insight into the kinetics can be gained from the analysis presented below. The isotopic exchange rate τ^{-1} can be written in terms of the general expressions

$${}^{-1} = (H_2O)^{-1}d(H_2O)/dt = ({}^{4}/{}_{3})k_1(H_2O) + ({}^{3}/{}_{3})k_2(H_3O^+) + ({}^{2}/{}_{3})k_3(SH^+) + k_4(OH^-) + k_5(H_2O)$$
(9)

The k_1 mechanism represents exchange by means of the autoprotolysis reaction of water

$$HOH + HO^*H^* \longrightarrow HOH^* + HO^*H$$
(10)

Eigen and de Maeyer³³ have determined an activation energy of 13.5 kcal./mole for this reaction in liquid water; a similar activation energy would be expected for the reaction in the solvents of interest here. But the temperature dependences reported in the preceding section put an upper limit of 1.7 kcal./mole on the effective activation energy for exchange in acetone³⁴ and an even lower one on those in nitromethane, acetonitrile and p-dioxane. This mechanism can be ruled out, therefore. The k_2 , k_3 and k_4 mechanisms all depend upon the concentration of ionic species and, therefore, upon $(K_W)^{1/2}$. This dependence implies a large effective activation energy since K_W is an exponential function of the endothermic ionization energy of H_2O ; the ionization enthalpy of H_2O in aqueous medium is 14 kcal./mole and it probably is larger in the other media studied. This large effective activation energy is incompatible with the lack of temperature variation reported in the preceding section; therefore, the ionic mechanisms probably are not important in the four "weakly basic solvents" considered here.

The k_5 mechanism represents direct proton exchange occurring within water polymers; k_5 might well be a function of (H₂O). In the discussion of water polymers above, it was stated that the trimer concentration probably varied as the square of the formal water concentration; therefore, if the principal exchange takes place within trimers, k_5 actually is approximately independent of the water concentration and the exchange is second order in (H₂O) as observed. A possible trimeric proton exchange mechanism could be a ring transfer such as



The rate of exchange of water molecules among monomers and polymers is probably very rapid, much more rapid than proton exchange, since only one species of proton is observed in the n.m.r. spectrum of H₂O diluted in organic solvents. As mentioned in an earlier section, the concentration of water trimers in solvents that are less basic than H₂O should have a small negative temperature dependence since the energy (enthalpy, ΔH_t) of trimer formation is negative. Proton transfer in the ring should require an activation energy E_r^* of

(33) M. Eigen and L. de Maeyer, Z. Elektrochem., 60, 1037 (1956). (34) τ^{-1} for acctone was measured over a smaller temperature range than were τ^{-1} 's in other solvents.

⁽³²⁾ S. Winstein, private communication.

a few kcal./mole. The effective activation energy for the k_5 mechanism is then E^*

$$E^* = E_r^* + \Delta H_t \tag{12}$$

and because of the negative ΔH_t , E^* may be very small. This mechanism is compatible with the experimental observations of the temperature dependence of τ^{-1} .

Mechanisms such as that in eq. 11 have been suggested by Grunwald and co-workers³⁵ and do not appear to be unique with the systems under study here. No serious attempt has been made to estimate theoretically the exchange rate of the process in eq. 11 but the relative rates for the weakly basic solvents can be estimated. If this trimeric process is the dominant one

$$\frac{1}{\tau} = k_{\mathfrak{s}}[(\mathrm{H}_{\mathfrak{s}}\mathrm{O})_{\mathfrak{s}}]/(\mathrm{H}_{\mathfrak{s}}\mathrm{O}) \tag{13}$$

and the relative rates, $\tau_{a}^{-1}/\tau_{b}^{-1}$, for two weakly basic solvents, are given by the relation

$$\frac{\tau_{\mathbf{a}}^{-1}}{\tau_{\mathbf{b}}^{-1}} = \frac{\left[(\mathrm{H}_{2}\mathrm{O})_{\mathbf{z}}\right]_{\mathbf{a}}}{\left[(\mathrm{H}_{2}\mathrm{O})_{\mathbf{z}}\right]_{\mathbf{b}}}$$
(14)

Although the concentrations of trimer given in Table II are uncertain, the ratios of trimer concentrations determined in Table II should be good estimates of the true trimer concentration ratios. If the proton exchange rate for H₂O in acetone is attributed entirely to this trimeric mechanism, the contribution of this mechanism to the exchange rate τ_5^{-1} in other solvents can be calculated by means of eq. 14. These are given in Table I. For the four most weakly basic solvents the theoretical τ_{5}^{-1} are in reasonably good agreement with experiment. For pyridine τ_5^{-1} increases but not sufficiently to account for the observed rate of exchange; however, it was pointed out earlier that the trimer concentration in pyridine is subject to more uncertainty than in the other solvents, and the k_{s} mechanism could, therefore, possibly explain the observed exchange rates in pyridine. For dimethyl sulfoxide τ_5^{-1} does not adequately explain the observed τ^{-1} .

The ionic reactions may well play a role in the more strongly basic solvents. The k_2 mechanism represents the reaction

$$H_3O^+ + HO^*H^* \longrightarrow H_2O + H^*O^*H_2^+$$
 (15)

and k₄ the reaction

$$OH^- + HO^*H^* \longrightarrow HOH^* + O^*H^-$$
 (16)

Recently Loewenstein and Szöke³⁶ have reported activation energies of 2.6 and 4.8 kcal./mole for reactions 15 and 16, respectively, in aqueous medium. Earlier work, both theoretical^{37,88} and experimental,^{39–41} also indicates small activation energies for these reactions. The rate constant k_2 can be estimated on the assumption that it is diffusion controlled^{41,42}

(35) E. Grunwald, private communication.

(36) A. Loewenstein and A. Szöke, J. Am. Chem. Soc., 84, 1151 (1962).

(37) B. Stearn and H. Byring, J. Chem. Phys., 5, 113 (1937).

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(41) M. T. Emerson, B. Grunwald and R. T. Kromhout, J. Chem. Phys., 33, 547 (1960).

(42) M. V. Smoluchowski, Z. physik. Chem., 113, 35 (1924).

$k_2 = 4\pi P(D_1 + D_2)\sigma N_0/2000$ (17)

where P is a steric factor, estimated to be $\binom{3}{4}\binom{1}{2}$ on the basis of the arguments used by Emerson and co-workers,⁴¹ N_0 is Avogadro's number, σ is the collision diameter of the two species which is about 3 Å., D_1 and D_2 are the diffusion coefficients for H₈O⁺ and H₂O, respectively, and a factor of (1/2) has been introduced to allow for the fact that the proton may be on either of the two oxygen atoms at the end of an encounter. In acetone $D_1 = 1.5 \times 10^{-5}$ and $D_2 = 2.06 \times 10^{-5}$ cm.²/sec. and, consequently, $k_2 = 1.6 \times 10^9$ liter/mole sec.⁴³ This rate constant together with the H_3O^+ concentration in Table III yields an exchange rate $\tau_2^{-1} = 0.03$ sec.⁻¹. Of course, the H₃O⁺ concentration is very approximate and may be in error by several orders of magnitude, but the small value of τ_2^{-1} is consistent with the discussion of activation energies presented above. Additional justification for the neglect of k_2 is obtained by noting that relatively large concentrations of acid, *i.e.*, in excess of 10^{-6} molar, are needed to increase the exchange rate appreciably. The same comments apply for all the solvents studied except nitromethane which can react with water to form H_3O^+ and $CH_2=NO_2^-$ with an equilibrium constant in water⁴⁴ of 6.3 \times 10⁻¹¹; if nitromethane rather than water is the solvent the equilibrium constant will be smaller. If the k_2 mechanism becomes significant because of this reaction, the temperature dependence of the rate constant would depend upon the temperature dependence of the equilibrium constant. The latter dependence has not been measured and it is not, therefore, known whether it is compatible with the measured activation energy for proton exchange. It was noted in the present work that a 1.1 formal solution of D_2O in nitromethane showed only about 20% conversion of D₂O to HDO after 72 hours at room temperature.

It has been shown that in liquid water k_2/k_4 is two to three^{36,45}; this ratio should be relatively insensitive to solvent effects since there is little change in the solvation of the two species during the formation of the transition state and since the solvent probably affects the diffusion rate of H₃O⁺ and OH^- about equally. If the ratio is invariant with change of solvent, τ_4^{-1} is probably negligible in nitromethane, acctone, acetonitrile and dioxane. For dimethyl sulfoxide a diffusion calculation yields $k_4 = 1.5 \times 10^8$ liter/mole sec. which, with the OH- concentration given in Table III, yields an exchange rate τ_4^{-1} which is much larger than the experimental value. The discrepancy between experimental and calculated exchange rates in dimethylsulfoxide may arise from errors in the estimation of the OH⁻ concentration or it may be due to an additional activation energy resulting from the presence of strong hydrogen bonds.³⁶ Nevertheless, it seems evident that the ionic k_4 mechanism may become important in the more strongly basic

⁽⁴³⁾ The diffusion coefficient for H_2O in acctone is known (Intern. Critical Tables, Vol. III). Values of D in other solvents were calculated by the Einstein relation which assumes that D is inversely proportional to the viscosity.

⁽⁴⁴⁾ R. P. Bell, "The Proton in Chemistry," Cornell University Press, Ithaca, New York, 1959, p. 53.

⁽⁴⁵⁾ S. Meiboom, J. Chem. Phys., 34, 375 (1961).

solvents. The calculated value of τ_4^{-1} for pyridine is 2.2 sec.⁻¹ and for triethylamine it will be even larger than for dimethylsulfoxide (see Table I).

The k_3 mechanism is represented by the reactions

$$SH^+ + HOH \xrightarrow{\longrightarrow} S + HOH_2^+$$
 (18)

One would expect the activation energy to be approximately the larger of the proton affinities of S and H₂O. Since the activation energy for the reaction in eq. 15 is about 2.5 kcal./mole, one might, in accordance with the discussion in the section on ionic concentrations, expect the activation energy for the k_3 mechanism to be approximately $[|2.3RTp\cdot(K_W/K_B)| + 2.5]$ kcal./mole. If S is NH₃ the activation energy is 12.2 kcal./mole.⁴⁰ In the strongly basic solvents the SH⁺ concentration is large but so is the activation energy for the k_3 mechanism is, therefore, probably not very important in the systems studied here.

Discussion of Kinetics.—The following have been offered as explanations for the proton exchange rates of water in organic solvents. For nitromethane, acetone, acetonitrile, dioxane and pyridine the H_2O trimeric mechanism is compatible with the kinetic data, *i.e.*, relative rates, low apparent activation energy, second order dependence on water concentration, and relative insensitivity to added acid. The trimer concentration is very low for dimethyl sulfoxide and triethylamine. The k_3 (SH⁺) mechanism probably has too high an activation energy to compete with the other process; $k_2(H_3O^+)$ is not too important because of the small H_3O^+ concentrations; $k_4(OH^-)$ is negligible for acetone, nitromethane, acetonitrile, dioxane and possibly for pyridine, because of the low OH⁻ concentrations. In the more basic solvents, dimethyl sulfoxide and triethylamine, where the OH⁻ concentration is large, $k_4(OH^-)$ probably becomes the dominant proton exchange term.

The technique used here for studying proton exchange rates differs from the usual method; instead of studying individual line broadening arising from the exchange of chemically distinct protons, the variation in spectral pattern, arising from the exchange of chemically similar deuterons and protons, has been analyzed. It should be noted that any appreciable kinetic isotope effect would have been observed by means of experiments in which the H/D ratio was varied. No such effect was measured.

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Appendix

The effect of exchange on the line shape has been treated both classically,^{46,47} using the Bloch equations, and by quantum mechanics.^{30,31,48} If the system is allowed to make random jumps between various sites ω_j , subject to the constraint that the probability P_j of the system's being at ω_j is independent of time, the spectral density $I(\omega)$ is given by by the formula of Sack³⁰

$$I(\omega) = Re\{\mathbf{P} \cdot (i\omega \mathbf{E}_n - i\Omega - \pi)^{-1}\mathbf{I}\}$$
(A-1)

where \mathbf{E}_n is a unit matrix of order n, 1 is a column matrix with all elements equal to unit, Ω is a diagonal matrix with elements $\Omega_{ij} = \omega_{\bar{i}}$, \mathbf{P} is a row matrix with elements P_i , and Π is a matrix with elements

$$\Pi_{jk} = p_{jk} \quad (j \neq k) \tag{A-2}$$

$$\Pi_{ij} = -\sum_{k \neq j} p_{ik} \qquad (A-2a)$$

and p_{ik} is the probability per unit time of a jump from ω_i to ω_2 , ω_3 and ω_4 corresponding to the species HDO ($m_D = -1$), HDO ($m_D = 0$), HDO ($m_D = 1$) and H₂O, respectively. If ω_2 is arbitrarily set equal to zero, ω_1 , ω_3 and ω_4 are located at $2\pi A_{\rm HD}$, $-2\pi A_{\rm HD}$ and $-2\pi \delta_{\rm HDO}$, respectively. Since the line shape depends upon the relative populations of the four sites, and the equilibrium constant for the formation of HDO from H₂O and D₂O is about 4, one can set $P_1 = P_2 =$ $P_3 = 1$ and $P_4 = 3(H/D)$ where H/D is the ratio of protons to deuterons in the mixture. The transition probabilities per unit time p_{ik} can be related to the mean time τ : for H/D = 1

$$p_{jk} = 1/6\tau \ k \neq 4$$
 (A-3)

$$p_{j4} = 1/2\tau \, j \neq 4$$
 (A-3a)

The spectral density can now be expressed as

$$I(\omega) = \frac{EF + GH}{E^2 + G^2}$$
(A-4)

where

 $E = ABCD - 12P^2 (AB + AC + 2AD + BC + 2BD + 2CD)$

 $F = 1296P^3 - 12P(2AB + 2AC + AD + 2BC + BD + CD)$

- $G = i[36P^{3} (A + B + C + 3D) 5P (0.6ABC + ABD + ACD + BCD)]$
- $H = i[36P^{2} (5A + 5B + 5C + 3D) (BCD + ACD + ABD + 3ABC)]$ (A-5)

in which

$$P = 1/6\tau$$

$$A = (\omega - \omega_1)$$

$$B = (\omega - \omega_2)$$

$$C = (\omega - \omega_3)$$

$$D = (\omega - \omega_4)$$
(A-6)

A FORTRAN program for the IBM 7090 computer was written to perform the numerical calculation. The resulting spectra were plotted as functions of $A_{\rm HD7}$ and $\delta_{\rm HD7}$ (see Fig. 5). Details are given in ref. 14.

If a kinetic isotope effect must be considered, τ^{-1} must be replaced by $\tau_{\rm H}^{-1} + \tau_{\rm D}^{-1}$ where $\tau_{\rm H}^{-1}$ and $\tau_{\rm D}^{-1}$ are the "jumping rates" for the proton and deuteron, respectively. In mixtures with (H/D) $\neq 1$ these terms must be appropriately.

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